

THE ENIAC FORECASTS

A Recreation

BY PETER LYNCH

NCEP–NCAR reanalyses help show that four historic forecasts made in 1950 with a pioneering electronic computer all had some predictive skill and, with a minor modification, might have been still better.



The first weather forecasts executed on an automatic computer were described in a landmark paper by Charney et al. (1950, hereafter CFvN). They used the Electronic Numerical Integrator and Computer (ENIAC), which was the most powerful computer available for the project, albeit primitive by modern standards. The results were sufficiently encouraging that numerical weather prediction became an operational reality within about five years.

CFvN subjectively compared the forecasts to analyses and drew general conclusions about their quality.

However, they were not verified objectively. In this study, we recreate the four forecasts using data available through the National Centers for Environmental Prediction–National Center for Atmospheric Research (NCEP–NCAR) 50-year reanalysis project. A comparison of the original and reconstructed forecasts shows them to be in good agreement. Quantitative verification of the forecasts yields surprising results: On the basis of root-mean-square errors, persistence beats the forecast in three of the four cases. The mean error, or bias, is smaller for persistence in all four cases. However, when S1 scores (Teweles and Wobus 1954) are compared, all four forecasts show skill, and three are substantially better than persistence.

FIG. 1. Visitors and some participants in the 1950 ENIAC computations. (left to right) Harry Wexler, John von Neumann, M. H. Frankel, Jerome Namias, John Freeman, Ragnar Fjørtoft, Francis Reichelderfer, and Jule Charney. (Provided by MIT Museum.)

PREPARING THE GROUND. John von Neumann was one of the leading mathematicians of

the twentieth century. He made important contributions in several areas: mathematical logic, functional analysis, abstract algebra, quantum physics, game theory, and the development and application of computers. In the mid-1930s von Neumann became interested in turbulent fluid flows. He saw that progress in hydrodynamics would be greatly accelerated if a means for solving complex equations numerically were available. It was clear that very fast automatic computing machinery was required. According to Thompson (1983), von Neumann regarded weather prediction by numerical means as “the most complex, interactive, and highly nonlinear problem that had ever been conceived of—one that would challenge the capabilities of the fastest computing devices for many years.” Indeed, weather forecasting has remained a grand challenge for computing ever since.

In May 1946, a proposal to establish the Meteorology Project at the Institute for Advanced Study (IAS) in Princeton, New Jersey, was successful in attracting funds from the U.S. Navy’s Office of Research and Inventions [the proposal is reprinted in Thompson (1983)]. A conference on meteorology was arranged at IAS the following August, and many of the leaders of the field attended. At the time, Jule Charney was visiting Rossby, who arranged for him to participate in the conference. Perhaps the most significant consequence of this conference was the opportunity for von Neumann to meet Charney. Von Neumann later persuaded Charney to lead the Meteorology Project, the primary goal of which was to investigate weather prediction by numerical means. The project ran from 1948 to 1956.

The initial plan was to integrate the primitive equations of the atmosphere, but the existence of high-speed gravity wave solutions meant that the volume of computation would exceed the capabilities of the available computers. The limitation, known as the Courant–Friedrichs–Lewy (CFL) criterion, restricts the maximum time step permitted for stable numerical integrations (Courant et al. 1928). There was also a more fundamental difficulty: the

impossibility of accurately calculating the divergence from the observations. The key paper, “On a physical basis for numerical prediction of large-scale motions in the atmosphere” (Charney 1949), addresses some crucially important issues. In this paper, Charney considered the means of dealing with high-frequency noise, proposing a hierarchy of filtered models. In his baroclinic instability study, Charney had derived a mathematically tractable equation for the unstable waves “by eliminating from consideration at the outset the meteorologically unimportant acoustic and shearing-gravitational oscillations” (Charney 1947). He realized that a general filtering principle was desirable. Such a system would have dramatic consequences for numerical integration. The time step dictated by the CFL criterion varies inversely with the speed of the fastest solution—fast gravity waves imply a very short time step, and the removal of these waves leads to a far less stringent limitation, allowing a much larger time step to be used.

An account of the filtered system was published in the paper “On the scale of atmospheric motions” (Charney 1948); this paper was to have a profound impact on the subsequent development of dynamic meteorology. Charney analyzed the primitive equations using the technique of scale analysis. He was able to simplify the system in such a way that the gravity wave solutions were completely eliminated. The resulting equations are known as the *quasi-geostrophic* equations. The system boils down to a single prognostic equation for the quasigeostrophic potential vorticity. All that is required by way of initial data to solve this equation is a knowledge of the three-dimensional pressure field (and appropriate boundary conditions).

In the special case of horizontal flow with constant static stability, the vertical variation can be separated out and the quasigeostrophic potential vorticity equation reduces to the nondivergent barotropic vorticity equation [BVE; see appendix A, (A1)]. This equation represents the conservation of absolute vorticity, the sum of the vorticity of the flow, and the vorticity resulting from the Earth’s spin. The barotropic equation had, of course, been used by Rossby in his analytical study of atmospheric waves (Rossby 1939), but the scientific view was that it was incapable of producing a quantitatively accurate prediction of atmospheric flow. We will describe the pioneering achievement of the group that carried out the first numerical integration of this simple equation, and will present the results of repeating the forecasts using initial data derived from the NCEP–NCAR 50-year reanalysis.

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INTO ACTION. The original integrations, which were the first computer weather forecasts ever made, are described in the much-cited paper of CFvN. This paper gives a complete account of the computational algorithm and discusses four forecast cases, all from initial data in 1949. The paper caused quite a stir when it appeared. A close study of it is recommended to readers wishing to have a deeper understanding of this pioneering work, because we must omit many details.

The authors outlined their reasons for starting with the barotropic equation as follows: the large-scale motions of the atmosphere are predominantly barotropic; the simple model could serve as a valuable pilot study for more complex integrations; and, if the results proved to be sufficiently accurate, barotropic forecasts could be utilized in an operational context. In fact, few if any people anticipated the enormous practical value of this simple model and the leading role it was to play in operational prediction for many years to come (Platzman 1979).

The ENIAC, which had been completed in 1945, was the first multipurpose electronic digital computer ever built. It was installed at the U.S. Army's Ballistics Research Laboratories at Aberdeen, Maryland. ENIAC was gigantic, weighing 30 tons, with 18,000 thermionic valves, massive banks of switches, and large plug boards with tangled skeins of connecting wires, filling a large room and consuming some 140 kW of power. Program commands were specified by setting the positions of a multitude of 10-pole rotary switches on large arrays called function tables, and input and output were generated by means of punch cards. The time between machine failures was typically a few hours, making the use of the computer a wearisome task for those operating it. McCartney (1999) provides an absorbing account of the origins, design, development, and destiny of ENIAC. Figure 1 shows some of the key personalities associated with the first computer forecasts.

The derivation of the basic prediction equation and the numerical algorithm used to solve it are fully described in appendix A. The equation chosen by CFvN was formulated using the geopotential height as the prognostic variable. The advection term was expressed as a Jacobian. The lateral boundary conditions

required to solve the BVE were investigated. It transpires that, to determine the motion, it is necessary and sufficient to specify the height on the whole boundary and the vorticity over that part where the flow is inward.

Initial data for the forecasts were taken from the manual 500-hPa analysis of the U.S. Weather Bureau, discretized to a grid of 19×16 points (Fig. 2). The grid interval was 736 km at the North Pole (494 km at 20°N), corresponding to 8° longitude at 45°N . Centered spatial finite differences and a leapfrog time scheme were used. The boundary heights were held constant, at their initial values, throughout each 24-h integration. The BVE gives an expression for the rate of change of the Laplacian of geopotential height in terms of the advection (the Jacobian term). Once this quantity is calculated, the tendency of the height field is obtained by solving a Poisson equation with homogeneous boundary conditions. The height may then be advanced to the next time level. This cycle may be repeated as often as required.

As the construction of von Neumann's computer at IAS was delayed, permission was obtained to use ENIAC. This was arranged through the offices of Francis Reichelderfer, Chief of the Weather Bureau. The story of the mission to Aberdeen was recounted by George Platzman (1979) in his Victor P. Starr Memorial Lecture. The venture began on 5 March 1950 when "an eager band of five meteorologists

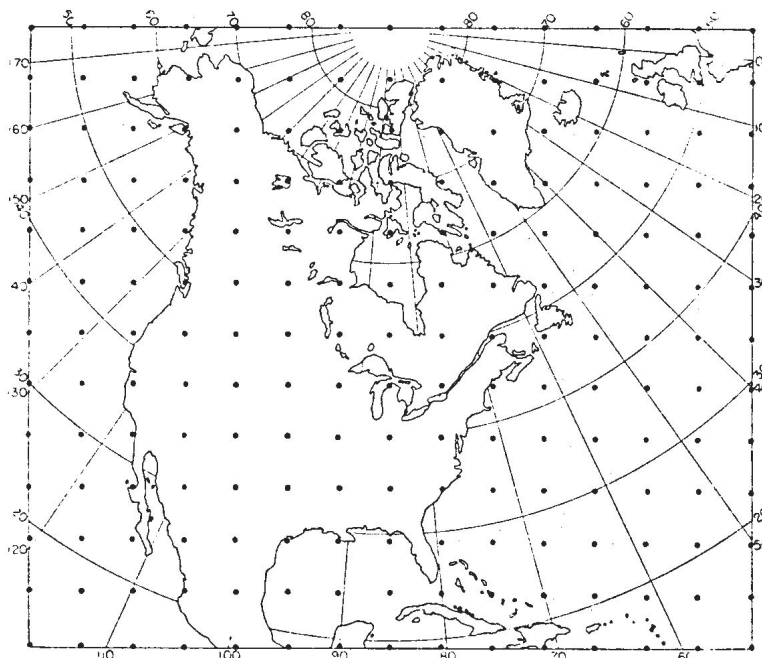


FIG. 2. Computation grid used for the ENIAC forecasts. One line is omitted from the southern edge and two lines from the remaining edges (from CFvN).

arrived in Aberdeen, Maryland, to play their roles in a remarkable exploit.” The five players were Jule Charney, Ragnar Fjørtoft, John Freeman, George Platzman, and Joseph Smagorinsky. The work continued for 33 days and nights, with only brief interruptions. The trials and tribulations of this intrepid troupe were described in Platzman’s lecture. There were the usual blunders familiar to programmers. The difficulties were exacerbated by the primitive machine language, the requirement to set numerous switches manually, the assignment of scale factors necessitated by the fixed-point nature of ENIAC, and the tedious and intricate card-deck operations (about 100,000 cards were punched during

the month). Despite these difficulties, the expedition ended in triumph. Four 24-h forecasts were made, and the results clearly indicated that the large-scale features of the midtropospheric flow could be forecast barotropically with a reasonable resemblance to reality.

The forecast starting at 0300 UTC 5 January 1949 is shown in Fig. 3. Figure 3a is the analysis of 500-hPa geopotential and absolute vorticity, based on the U.S. Weather Bureau manual analysis. Figure 3b is the corresponding analysis 24 h later. The forecast height and vorticity are shown in Fig. 3d. This prediction was, according to CFvN, “uniformly poor.” The feature of primary interest was an intense depression over the

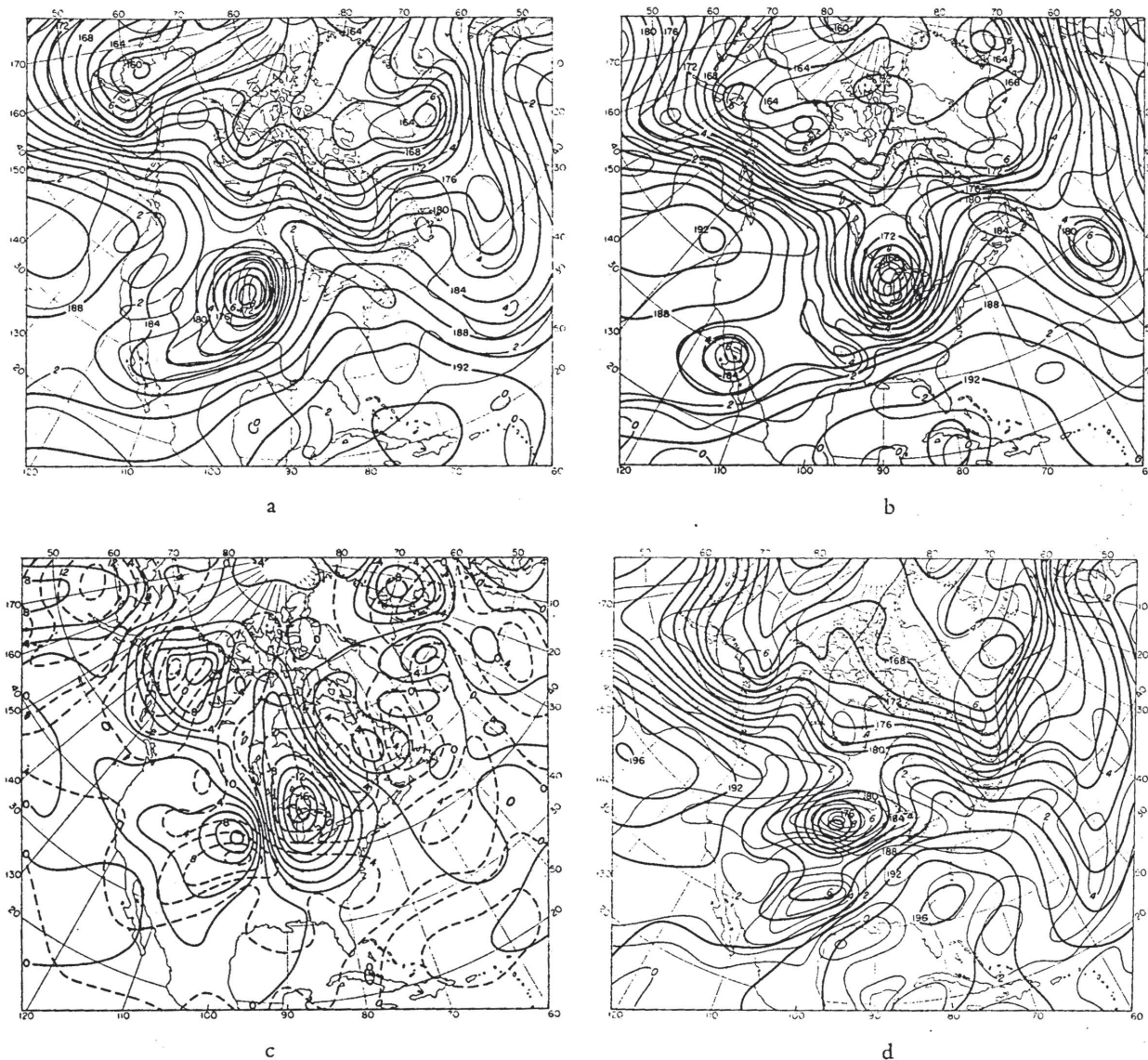


FIG. 3. Forecast of CFvN from 5 Jan 1949. (a) Analysis of 500-hPa geopotential height (thick lines) and absolute vorticity (thin lines) for 0300 UTC 5 Jan. (b) Analysis for 0300 UTC 6 Jan. (c) Observed change (solid) and forecast change (dashed). (d) Forecast height and vorticity valid at 0300 UTC 6 Jan (from CFvN). Height units are hundreds of feet, contour interval is 200 ft. Vorticity units and contour interval (10^{-5} s^{-1}).

United States. This deepened and moved northeast to the 90°W meridian in 24 h. The forecast displacement was too slow and the shape of the depression was distorted. Also, the low center over the Gulf of California in the verifying analysis is absent in the forecast (but, as we will see, it is also absent in the reanalysis).

CFvN considered the causes of the errors in their forecasts, and attempted to distinguish between the following various factors: spatial truncation, limitations of the model formulation, and the effects of baroclinicity versus barotropy. We will not pursue this line; however, those interested may easily investigate the effects of truncation by rerunning the model with a higher spatial resolution; both the model and data are available (see appendix B).

THE NCEP–NCAR REANALYSIS. The initial fields for the four ENIAC forecasts were valid for 5 January, 30–31 January, and 13 February 1949, predating automated data assimilation and numerical prediction, so no objective analyses were produced in real time. Thus, when a reconstruction of the forecasts was first conceived, a laborious digitization of hand-drawn charts from the pre-NWP era appeared necessary. However, a retrospective global analysis of the atmosphere, covering more than 50 years, has been undertaken by NCEP and NCAR. Because the reanalysis extends back to 1948, it includes the period chosen for the ENIAC integrations. The NCEP–NCAR 50-year reanalysis is described by Kistler et al. (2001); it is a development from an earlier 40-year reanalysis (Kalnay et al. 1996). The products are freely available from NCEP, NCAR, and the National Oceanic and Atmospheric Administration (NOAA).

The reanalysis project used a three-dimensional variational data assimilation (3DVAR) scheme called spectral statistical interpolation (Parrish and Derber 1992). The spectral representation had a triangular truncation of 62 waves, corresponding to a horizontal resolution of about 210 km. Prior to the International Geophysical Year in 1957, upper-air observations were generally made at 0300 and 1500 UTC, and occasionally at 0900 and 2100 UTC. For this reason, the reanalysis for the first decade (1948–57) was performed at 0300, 0900, 1500, and 2100 UTC. This is ideal, because the initial time for each of the four ENIAC forecasts was 0300 UTC. The data are available on a $2.5^\circ \times 2.5^\circ$ grid. While relatively coarse by modern standards, this is far above the resolution used in the ENIAC runs, which had a grid spacing of about 8° longitude (at 45°N). The 500-hPa analyses were downloaded from the NCEP–NCAR

reanalysis Web site, converted from Gridded Binary (GRIB) to American Standard Code for Information Interchange (ASCII), and interpolated to the ENIAC grid.

RECREATING THE FORECASTS. At the time of the ENIAC experiment, computer programming was at an embryonic stage of development. Given the limitations of the hardware, with just a dozen or so internal registers for data storage, the solution of the equations had to be broken down into elementary, discrete steps. The results of each computation were punched onto cards, and these punch cards provided the inputs for the following steps. The programming was at the level of machine code; high-level languages such as FORTRAN were still in the future. In recreating the forecasts, we have the luxury of using powerful, user-friendly coding languages. The MATLAB system has been selected because it is simple to learn, is widely available, and has an embedded graphical system. (Free software similar to MATLAB is available; www.gnu.org/software/octave/.)

The CFvN paper provides a full description of the solution algorithm. The program `eniac.m` was constructed following the original algorithm precisely, including the specification of the boundary conditions and the Fourier transform solution method for the Poisson equation. Hence, given initial data identical to that used in CFvN, the recreated forecasts should be identical to those made in 1950. Of course, the reanalyzed fields are not identical to those originally used, and the verification analyses are also different. Thus, the expectation of an exact replication of the original forecasts is unrealistic. Nevertheless, we will see that the original and new results are very similar.

Figure 4 is the reconstructed forecast for 0300 UTC 6 January. The layout is as for Fig. 3. The main features of the analyses are in broad agreement with the originals. Also, the forecast shows the inadequate speed of movement of the low center and its distortion, as in the original forecast. Despite their similarities, the original and reanalyzed fields have marked differences. Note, for example, the low center over the Gulf of California in the original verifying analysis (Fig. 3b), which is absent in the reanalysis. We may recall the dearth of upper-air observations in 1949, especially over the oceans, which resulted in significant uncertainties in the analysis.

Objective verification scores for the four reconstructed forecasts are presented in Table 1. There is a substantial bias error in all the forecasts; the mean

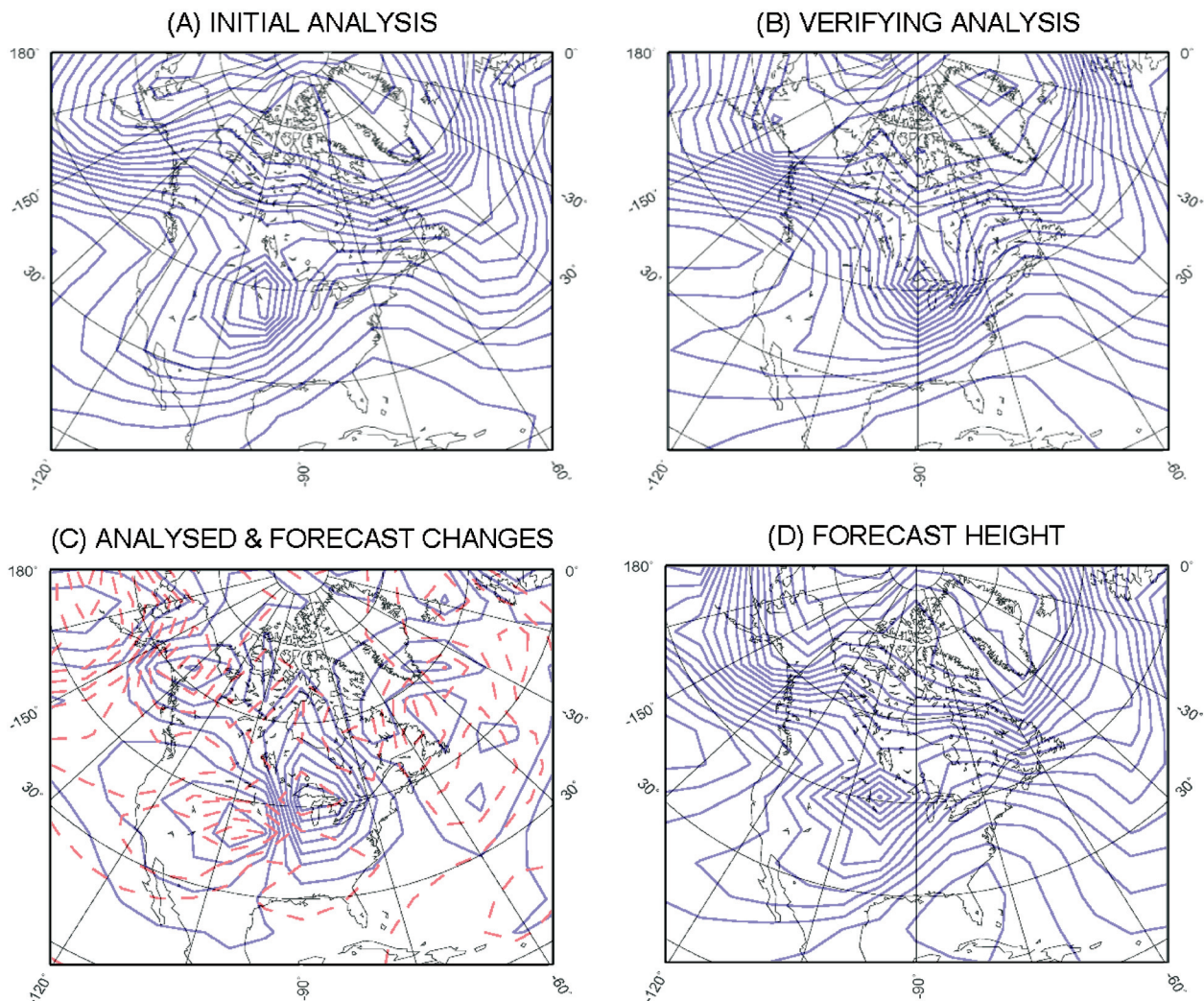


FIG. 4. Reconstructed forecast for 5 Jan 1949: (a) analysis of 500-hPa geopotential height (thick lines) for 0300 UTC 5 Jan, (b) analysis for 0300 UTC 6 Jan, (c) observed change (solid) and forecast change (dashed), and (d) forecast height valid at 0300 UTC 6 Jan. Height contour interval is 50 m.

errors greatly exceed those of a persistence forecast. The root-mean-square (RMS) error is greater than persistence in case 1, less in case 2, and about equal in the remaining cases. The S1 score, introduced at an early stage of NWP (Teweles and Wobus 1954), measures skill in predicting gradients that are meteorologically significant. The S1 score is about the same as that of persistence for case 1 and significantly better for the other three cases. On the basis of this score, all four forecasts have some skill.

In CFvN it is noted that the computation time for a 24-h forecast was about 24 h, that is, the team could just keep pace with the weather provided ENIAC did not fail. This time included offline operations: reading, punching, and interfiling punch cards. Von Neumann estimated that it would have taken five hand-computer years to duplicate the ENIAC com-

putations (Nebeker 1995). Addressing the National Academy of Sciences, Charney said, “It mattered little that the twenty-four-hour prediction was twenty-four hours in the making. That was purely a technological problem. Two years later we were able to make the same prediction on our own machine [von Neumann’s IAS computer] in five minutes” (Charney 1955).

During his Starr Lecture, George Platzman arranged with IBM to repeat one of the ENIAC forecasts. The algorithm of CFvN was coded on an IBM 5110, a desktop machine then called a portable computer or “PC” (having a tiny fraction of the power of a modern PC). The program execution was completed within the hour or so of Platzman’s lecture. This implies a 24-fold speedup over the best rate achievable for ENIAC. The program eniac.m

was run on a Sony Vaio (model VGN-TX2XP) with MATLAB version 6. The main loop of the 24-h forecast ran in about 30 ms. One may question the precise significance of the time ratio—about three million to one—but it certainly indicates the dramatic increase in computing power over the past half-century.

INTO OPERATIONS. In an interview with George Platzman, Charney said “I think we were all rather surprised that the predictions were as good as they were” (Platzman 1990). Reviewing Charney’s influence on meteorology some years later, Norman Phillips wrote that the success of the ENIAC integrations was so clear that “the idea of numerical prediction began to be accepted immediately by the meteorological community” (Phillips 1990). The encouraging initial results of the Princeton team generated widespread interest and raised expectations that operationally useful computer forecasts would soon be a reality. Within a few years research groups were active in several universities and national weather services. The striking success of the barotropic forecasts had come as a surprise to most meteorologists. The barotropic vorticity equation simply states that the absolute vorticity of a fluid parcel is constant along its trajectory; the equation seemed too idealized to have any potential for operational use. The richness and power encapsulated in its nonlinear advection were greatly underestimated. Evidence that even the rudimentary barotropic model was capable of producing forecasts comparable in accuracy to those produced by conventional manual means rapidly accumulated. Indeed, baroclinic models used for early NWP operations were found to perform poorly, and the single-level model was reintroduced. The humble barotropic vorticity equation continued to provide useful guidance for almost a decade.

A SIMPLE ENHANCEMENT. A commemorative symposium was held in Potsdam, Germany, in March 2000 to mark the 50th anniversary of NWP. In his address at this gathering, Norman Phillips expressed the view that quasigeostrophic models might have been more productive if a streamfunction had been used in place of the geopotential (Phillips 2000). Use of the streamfunction form avoids the omission of a term involving the gradient of the Coriolis parameter (the beta term), yielding a more accurate equation (see appendix A). Phillips had good reason for his claim: much earlier, he had rerun a forecast of the famous Thanksgiving storm, a major cyclonic development in the eastern United States in

TABLE 1. Mean error (bias), RMS error (m), and SI score for the four forecasts (Fcst) and for corresponding persistence forecasts (Pers). In each case, bold type indicates whether the forecast or persistence is better.

Case	Mean error		RMS error		SI score	
	Fcst	Pers	Fcst	Pers	Fcst	Pers
1	56	-9	113	95	61	62
2	31	6	99	115	46	63
3	-35	20	93	89	46	58
4	39	1	82	81	39	50

November 1950. This storm became a popular case study, because it brought unusually severe weather and had not been well predicted in real time.¹ It was believed that baroclinic processes, which are not represented in the single-level model, were an essential factor in the rapid development of the storm. Phillips found, using a two-level filtered model, that there were large errors associated with the excess of the geostrophic wind over the true wind and that the streamfunction formulation resulted in a marked improvement in the forecast. In that paper (Phillips 1958), published in *Festschrift* for the 60th birthday of Erik Palmén, Phillips remarked that operational experience with the streamfunction equation “has appreciably improved the forecasts.” In a similar vein, George Cressman, the first director of the Joint Numerical Weather Prediction Unit, established in 1954, recalled that the spurious distortion of the flow in low latitudes was partly resolved by using the streamfunction equation (Cressman 1996). Thus, it is of interest to rerun the four ENIAC integrations using this equation and then compare them to the original forecasts.

The four forecasts were repeated using the streamfunction equation [appendix A, (A5)]; the initial streamfunction was obtained by dividing the geopotential height by a constant mean-latitude value of the Coriolis parameter. The verification scores are given in Table 2. The bias is more negative in all cases and substantially smaller in all but case 3. The RMS error is smaller in all cases. The SI score, averaged over all four cases, has improved by about a point. We conclude from this admittedly limited evidence that the streamfunction form of the BVE is superior.

¹ The Thanksgiving storm has been successfully predicted using a modern NWP system and data from the NCEP–NCAR 50-year reanalysis, yielding a remarkably accurate 4-day forecast (Kistler et al. 2001).

TABLE 2. Mean error (bias), RMS error (m), and SI score for the four forecasts using the height equation (*z* EQN) equation and the streamfunction equation (*ψ* EQN). In each case, bold type indicates which forecast is better.

Case	Mean error		RMS error		SI score	
	<i>z</i> EQN	<i>ψ</i> EQN	<i>z</i> EQN	<i>ψ</i> EQN	<i>z</i> EQN	<i>ψ</i> EQN
1	56	44	113	107	61	61
2	31	23	99	89	46	44
3	-35	-40	93	88	46	45
4	39	20	82	72	39	37

The pioneers who performed the ENIAC integrations might have used this equation and obtained better results. But, the initial data were for heights, and this must have seemed a more natural choice of variable. The determination of the initial streamfunction via the crude geostrophic relationship would not have appeared promising, and a more accurate relationship between the mass and wind fields, the nonlinear balance equation, would have been difficult to solve on ENIAC. Moreover, CFvN did not have the luxury of testing alternative formulations without considerable effort.

EPILOGUE: DOROTHY'S VIEW. CFvN were greatly inspired by the earlier attempt by Lewis Fry Richardson to predict atmospheric changes (Richardson 1922). Richardson's experiment produced outlandish results, for reasons that are examined in depth by Lynch (2006). It is gratifying that Richardson was made aware of the success in Princeton; Charney sent him copies of several reports, including the paper on the ENIAC integrations (Platzman 1968). Richardson congratulated Charney and his collaborators "on the remarkable progress which has been made in Princeton; and on the prospects for further improvement which you indicate." He then described a "tiny psychological experiment" on the diagrams in the *Tellus* paper, which he had performed with the help of his wife Dorothy. For each of the four forecasts, he asked her opinion as to whether a prediction of persistence was better or worse than the numerical prediction. Dorothy's view, tabulated in detail in Platzman (1968), was that the numerical prediction was, on average, better, though only marginally. The case that Dorothy identified as definitely better was the forecast starting from the 31 January analysis. This was also identified by Charney as "surprisingly good" (Platzman 1979). Richardson concluded that the ENIAC results were "an enormous scientific advance" on his own forecast, which was completely unrealistic. Indeed, they were not

only an advance, but the beginning of an onward march that is continuing to this day.

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APPENDIX A. SOLUTION OF THE BAROTROPIC VORTICITY EQUATION. The method chosen by CFvN to solve the barotropic vorticity equation,

$$\frac{d(\zeta + f)}{dt} = \frac{\partial \zeta}{\partial t} + \mathbf{V} \cdot \nabla(\zeta + f) = 0, \quad (\text{A1})$$

was based on using geopotential height as the prognostic variable.^{A1} If the wind is taken to be both geostrophic and nondivergent, we have

$$\mathbf{V} = (g/f)\mathbf{k} \times \nabla z; \mathbf{V} = \mathbf{k} \times \nabla \psi.$$

The vorticity is given by $\zeta = \nabla^2 \psi$. These relationships lead to the linear balance equation

$$\zeta = \nabla \cdot (g/f) \nabla z = (g/f) \nabla^2 z + \beta u/f. \quad (\text{A2})$$

CFvN ignored the β term, which can be shown by scaling arguments to be small. They then expressed the advection term as a Jacobian,

$$\begin{aligned} \mathbf{V} \cdot \nabla \alpha &= -\frac{g}{f} \frac{\partial z}{\partial y} \frac{\partial \alpha}{\partial x} + \frac{g}{f} \frac{\partial z}{\partial x} \frac{\partial \alpha}{\partial y} \\ &= -\frac{g}{f} J(\alpha, z). \end{aligned} \quad (\text{A3})$$

Now, using (A2) and (A3) in (A1), they arrived at

$$\frac{\partial}{\partial t} (\nabla^2 z) = J \left(\frac{g}{f} \nabla^2 z + f, z \right). \quad (\text{A4})$$

This was taken as their basic equation [Eq. (8) in CFvN]. It is interesting to observe that, had they

^{A1} All mathematical notation in this appendix is in accordance with standard meteorological usage and is consistent with CFvN.

chosen the streamfunction rather than the geopotential as the dependent variable, they could have used the equation

$$\frac{\partial}{\partial t}(\nabla^2 \psi) = J(\nabla^2 \psi + f, \psi), \quad (\text{A5})$$

thereby avoiding the neglect of the β term in (A2). The boundary conditions required to solve (A4) were investigated. It transpires that to determine the motion it is necessary and sufficient to specify z on the whole boundary and ζ over that part where the flow is inward.

The vorticity equation was transformed to a polar stereographic projection; this introduces a map factor $m = 2/(1 + \sin\varphi)$, where φ is the latitude (Haltiner and Williams 1980, 10–14). The Laplacian and Jacobian operators on the map and on the globe are related through this factor,

$$\nabla_{\text{globe}}^2 = m^2 \nabla_{\text{map}}^2 \quad \text{and} \quad J_{\text{globe}} = m^2 J_{\text{map}}.$$

Initial data were taken from the manual 500-hPa analysis of the U.S. Weather Bureau, discretized to a grid of 19×16 points (Fig. 2) with a grid interval corresponding to 8° longitude at 45°N (736 km at the North Pole and 494 km at 20°N). Centered spatial finite differences and a leapfrog time scheme were used. The boundary conditions were held constant throughout each 24-h integration. Equation (A4) is equivalent to the system

$$\frac{\partial \xi}{\partial t} = J\left(\frac{gm^2}{f}\xi + f, z\right), \quad (\text{A6})$$

$$\nabla^2 \frac{\partial z}{\partial t} = \frac{\partial \xi}{\partial t}, \quad (\text{A7})$$

where $\xi = \nabla^2 z$. The tendency of ξ is computed from (A6). Then, the Poisson equation [(A7)] is solved with homogeneous boundary conditions for the tendency of z , after which z and ξ are updated to the next time level. This cycle may be repeated as often as required. Time steps of 1, 2, and 3 h were all tried; with such a coarse spatial grid, even the longest time step produced stable integrations. For all forecast reconstructions, a time step of 1 h was used.

The Poisson equation [(A7)] was solved using a direct method devised by von Neumann. This method was more suited to the ENIAC than iterative relaxation methods, such as that of Richardson. Each step required four one-dimensional Fourier transforms. For a grid of $(K + 1) \times (L + 1)$ points, the solution may be written as

$$\left(\frac{\partial z}{\partial t}\right)_{mn} = -\left(\frac{\Delta s^2}{KL}\right) \sum_{i=1}^{K-1} \sum_{j=1}^{L-1} \sum_{k=1}^{K-1} \sum_{l=1}^{L-1} \left[\frac{\sin \frac{ik\pi}{K} \sin \frac{jl\pi}{L} \sin \frac{km\pi}{K} \sin \frac{ln\pi}{L}}{\sin^2 \frac{k\pi}{2K} + \sin^2 \frac{l\pi}{2L}} \right] \left(\frac{\partial \xi}{\partial t}\right)_{ij} \quad (\text{A8})$$

(complete details may be found in CFvN). For the integration of (A5), the initial streamfunction was defined as $\psi = gz/f_0$, where f_0 is the Coriolis parameter evaluated at 45°N .

APPENDIX B. STRUCTURE OF THE MATLAB PROGRAM ENIAC.M.

CFvN provide a full description of the solution algorithm. The data-handling and computing operations involved for each time step are shown schematically in Fig. B1 (from Platzman 1979). Each row indicates a program specification by setting upward of 5,000 switches (column 1), computing (column 2), punching out cards (column 3), and manipulating cards on offline equipment (column 4). Fourteen punch card operations were required for each time step because the internal memory of ENIAC was limited to 10 registers. The first row of Fig. B1 represents a step forward in time; the next depicts the computation of the Jacobian; then follow four Fourier transforms, corresponding to the four-fold summation in (A8). The final row indicates housekeeping computations and manipulations in preparation for the next step.

The program eniac.m was constructed following the original algorithm of CFvN precisely, including the specification of the boundary conditions and the Fourier transform solution method for the Poisson equation. The main steps in the solution algorithm are given below; the order of operations corresponds broadly to the program eniac.m (which includes documentation in the form of copious inline comments):

- 1) The forecast case is specified. Parameters that determine the space and time discretization are given. The Coriolis parameter and map factor are computed.
- 2) The sine function matrices for the Poisson equation solver are computed.

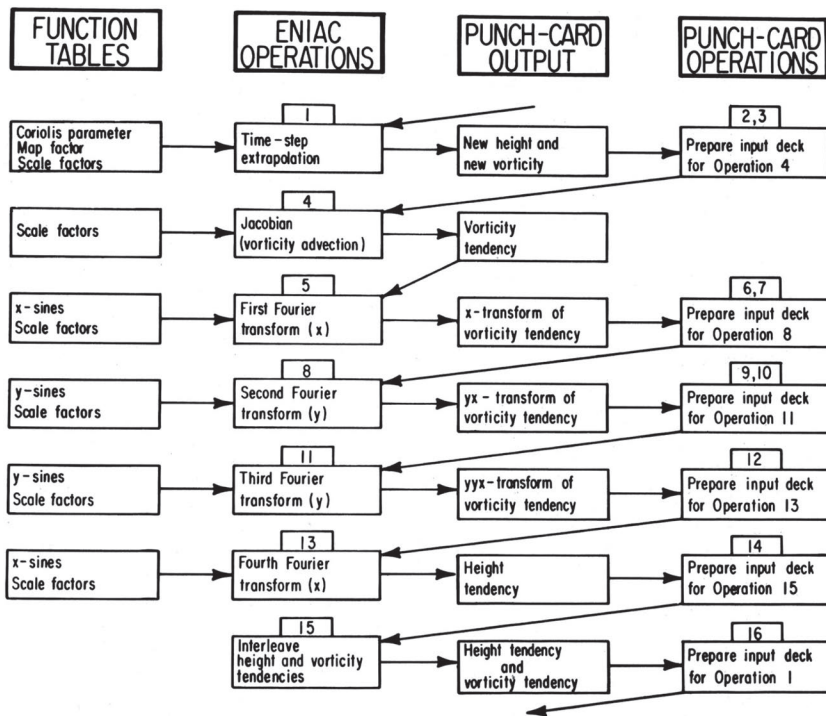


Fig. B1. Flowchart showing the 16 operations required for each time step of the ENIAC forecast (from Platzman 1979).

- 3) The initial and verifying analyses are read in and plotted.
- 4) The main time loop begins.
 - a) Spatial derivatives are computed by finite differences. The vorticity and Jacobian are determined. The tendency of the Laplacian of height ξ is given by the Jacobian.
 - b) Energy and enstrophy are calculated (nonessential diagnostics).
 - c) The updated values of ξ at the lateral boundaries are computed.
 - d) The values of ξ and of z at the next time step are calculated using the leapfrog method.
 - e) Intermediate forecast height fields may be plotted.
 - f) Housekeeping operations complete the time step.
- 5) Energy and enstrophy are plotted (for diagnostic purposes).
- 6) The forecast height field is plotted. Forecast and observed height changes are computed and plotted in a format similar to that used in CFvN.
- 7) Verification scores are computed.

The MATLAB code of `eniac.m` is available on the author's Web site (<http://mathsci.ucd.ie/~plynch/eniac/>). The relevant analyses, on a global $2.5^\circ \times 2.5^\circ$ grid are also obtainable there, together with a program

`PrepICs.m` to interpolate them to a polar stereographic grid. Maps of the four original and recreated forecasts are also available, along with miscellaneous supplementary material relating to the ENIAC integrations.

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